AGENDA

1. Preference Learning Tasks (Eyke)
2. Loss Functions (Johannes)
3. Preference Learning Techniques (Eyke)
   a. Learning Utility Functions
   b. Learning Preference Relations
   c. Structured Output Prediction
   d. Model-Based Preference Learning
   e. Local Preference Aggregation
4. Complexity of Preference Learning (Johannes)
5. Conclusions
Two Ways of Representing Preferences

- **Utility-based approach:** Evaluating single alternatives
  
  \[ U : \mathcal{A} \rightarrow \mathbb{R} \]

- **Relational approach:** Comparing pairs of alternatives
  
  \[ a \succeq b \iff a \text{ is not worse than } b \]
  
  weak preference

  \[ a \succ b \iff (a \succeq b) \land (b \not\succeq a) \]
  
  strict preference

  \[ a \sim b \iff (a \succeq b) \land (b \succeq a) \]
  
  indifference

  \[ a \perp b \iff (a \not\succeq b) \land (b \not\succeq a) \]
  
  incomparability
Utility Functions

- A utility function assigns a utility degree (typically a real number or an ordinal degree) to each alternative.
- Learning such a function essentially comes down to solving an (ordinal) regression problem.
- Often additional conditions, e.g., due to bounded utility ranges or monotonicity properties (→ learning monotone models)

- A utility function induces a ranking (total order), but not the other way around!
- But it can not represent a partial order!
- The feedback can be direct (exemplary utility degrees given) or indirect (inequality induced by order relation):

\[(x, u) \Rightarrow U(x) \approx u, \quad x \succ y \iff U(x) > U(y)\]

- direct feedback
- indirect feedback
Predicting Utilities on Ordinal Scales

(Graded) multilabel classification

<table>
<thead>
<tr>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>0</td>
<td>10</td>
<td>174</td>
<td>--</td>
<td>+</td>
<td>++</td>
<td>0</td>
</tr>
<tr>
<td>1.45</td>
<td>0</td>
<td>32</td>
<td>277</td>
<td>0</td>
<td>++</td>
<td>--</td>
<td>+</td>
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<tr>
<td>1.22</td>
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<td>46</td>
<td>421</td>
<td>--</td>
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<td>0</td>
<td>+</td>
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<tr>
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<td>165</td>
<td>0</td>
<td>+</td>
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<td>++</td>
</tr>
<tr>
<td>0.95</td>
<td>1</td>
<td>72</td>
<td>273</td>
<td>+</td>
<td>0</td>
<td>++</td>
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</tr>
<tr>
<td>1.04</td>
<td>0</td>
<td>33</td>
<td>158</td>
<td>+</td>
<td>+</td>
<td>++</td>
<td>--</td>
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</tbody>
</table>

Collaborative filtering

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>...</th>
<th>P38</th>
<th>...</th>
<th>P88</th>
<th>P89</th>
<th>P90</th>
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<tbody>
<tr>
<td>U1</td>
<td>1</td>
<td>4</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>3</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>U2</td>
<td>2</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>1</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U46</td>
<td>?</td>
<td>2</td>
<td>?</td>
<td>...</td>
<td>?</td>
<td>...</td>
<td>?</td>
<td>?</td>
<td>4</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U98</td>
<td>5</td>
<td>...</td>
<td>...</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U99</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exploiting dependencies (correlations) between items (labels, products, ...).

→ see work in MLC and RecSys communities
Learning Utility Functions from Indirect Feedback

- A (latent) utility function can also be used to solve ranking problems, such as instance, object or label ranking
  \[ \text{\rightarrow ranking by (estimated) utility degrees (scores)} \]

**Object ranking**

\[
\begin{align*}
(0.74, 1, 25, 165) & > (0.45, 0, 35, 155) \\
(0.47, 1, 46, 183) & > (0.57, 1, 61, 177) \\
(0.25, 0, 26, 199) & > (0.73, 0, 46, 185) \\
(0.95, 0, 73, 133) & > (0.25, 1, 35, 153) \\
(0.68, 1, 55, 147) & > (0.67, 0, 63, 182)
\end{align*}
\]

Find a utility function that agrees as much as possible with the preference information in the sense that, for most examples,

\[ x_i \succ y_i \iff U(x_i) > U(y_i) \]

**Instance ranking**

\[
\begin{array}{cccc|c}
X1 & X2 & X3 & X4 & \text{class} \\
0.34 & 0 & 10 & 174 & -- \\
1.45 & 0 & 32 & 277 & 0 \\
1.22 & 1 & 46 & 421 & -- \\
0.74 & 1 & 25 & 165 & ++ \\
0.95 & 1 & 72 & 273 & +
\end{array}
\]

Absolute preferences given, so in principle an ordinal regression problem. However, the goal is to maximize ranking instead of classification performance.
Ranking versus Classification

A ranker can be turned into a classifier via thresholding:

\[ f(x) > t \quad \text{positive} \quad f(x) < t \quad \text{negative} \]

A good classifier is not necessarily a good ranker:

- 2 classification but 10 ranking errors

⇒ learning **AUC-optimizing scoring classifiers**!
RankSVM and Related Methods (Bipartite Case)

- The idea is to minimize a convex upper bound on the empirical ranking error over a class of (kernalized) ranking functions:

\[
f^* \in \arg \min_{f \in \mathcal{F}} \left\{ \frac{1}{|P| \cdot |N|} \sum_{x \in P} \sum_{x' \in N} L(f, x, x') + \lambda \cdot R(f) \right\}
\]

regularizer

convex upper bound on

\[ \mathbb{1}(f(x) < f(x')) \]
The bipartite RankSVM algorithm [Herbrich et al. 2000, Joachimes 2002]:

\[ f^* \in \arg \min_{f \in F_K} \left\{ \frac{1}{|P| \cdot |N|} \sum_{x \in P} \sum_{x' \in N} (1 - (f(x) - f(x')))_+ + \frac{\lambda}{2} \cdot \|f\|^2_K \right\} \]

- The hinge loss
- The regularizer
- The reproducing kernel Hilbert space (RKHS) with kernel \( K \)

→ learning comes down to solving a QP problem
The bipartite RankBoost algorithm [Freund et al. 2003]:

\[
\begin{align*}
    f^* & \in \arg \min_{f \in \mathcal{L}(\mathcal{F}_\text{base})} \left\{ \frac{1}{|P| \cdot |N|} \sum_{x \in P} \sum_{x' \in N} \exp \left( -(f(x) - f(x')) \right) \right\} \\
\end{align*}
\]

→ learning by means of boosting techniques
Learning Utility Functions for Label Ranking

Label ranking is the problem of learning a function $\mathcal{X} \rightarrow \Omega$, with $\Omega$ the set of rankings (permutations) of a label set $\mathcal{Y} = \{y_1, y_2, \ldots, y_k\}$, from exemplary pairwise preferences $y_i \succ y_j$.

Can be tackled by learning utility functions $U_1(\cdot), \ldots, U_k(\cdot)$ that are as much as possible (but not too much) in agreement with the preferences in the training data. Given a new query $\mathbf{x}$, the labels are ranked according to utility degrees, i.e., a permutation $\pi$ is predicted such that

$$U_{\pi^{-1}(1)}(\mathbf{x}) > U_{\pi^{-1}(2)}(\mathbf{x}) > \ldots > U_{\pi^{-1}(k)}(\mathbf{x})$$
Label Ranking: Reduction to Binary Classification [Har-Peled et al. 2002]

Proceeding from linear utility functions

\[ U_i(x) = w_i \times x = (w_{i,1}, w_{i,2}, \ldots, w_{i,m})(x_1, x_2, \ldots, x_m)^\top, \]

a binary preference \( y_i \succ x y_j \) is equivalent to

\[ U_i(x) > U_j(x) \iff w_i \times x > w_j \times x \iff (w_i - w_j) \times x > 0 \]

and can be modeled as a linear constraint

\[ (w_1, w_2 \ldots w_k) \times (0 \ldots 0 x 0 \ldots 0 - x 0 \ldots 0)^\top > 0 \]

(\( m \times k \))-dimensional weight vector positive example in the new instance space

→ each pairwise comparison is turned into a binary classification example in a high-dimensional space!
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Learning Binary Preference Relations

- Learning **binary preferences** (in the form of predicates $P(x,y)$) is often simpler, especially if the training information is given in this form, too.
- However, it implies an additional step, namely **extracting a ranking** from a (predicted) preference relation.
- This step is not always trivial, since a predicted preference relation may exhibit inconsistencies and may not suggest a unique ranking in an unequivocal way.

$$
\begin{array}{c|cccc}
  f_{i,j} & y_1 & y_2 & y_3 & y_4 & y_5 \\
y_1 & 1 & 1 & 0 & 0 \\
y_2 & 0 & 0 & 1 & 0 \\
y_3 & 0 & 1 & 0 & 0 \\
y_4 & 1 & 0 & 1 & 1 \\
y_5 & 1 & 1 & 1 & 0 \\
\end{array}
$$

**inference**

$y_4 \succ y_5 \succ y_1 \succ y_3 \succ y_2$
Object Ranking: Learning to Order Things [Cohen et al. 99]

- In a first step, a **binary preference function** \( \text{PREF} \) is constructed; \( \text{PREF}(x,y) \in [0,1] \) is a measure of the certainty that \( x \) should be ranked before \( y \), and \( \text{PREF}(x,y) = 1 - \text{PREF}(y,x) \).

- This function is expressed as a linear combination of base preference functions:

\[
\text{PREF}(x, y) = \sum_{i=1}^{N} w_i \cdot R_i(x, y)
\]

- The weights can be learned, e.g., by means of the weighted majority algorithm [Littlestone & Warmuth 94].

- In a second step, a total order is derived, which is a much as possible in agreement with the binary preference relation.
Object Ranking: Learning to Order Things [Cohen et al. 99]

- The weighted feedback arc set problem: Find a permutation $\pi$ such that
  $$\sum_{(x, y): \pi(x) > \pi(y)} \text{PREF}(x, y)$$

becomes minimal.

\[
\begin{array}{c}
0.1 \\
0.6 \\
0.5 \\
0.8 \\
0.9 \\
0.7 \\
0.6 \\
0.6 \\
0.6 \\
0.8 \\
0.5 \\
0.3 \\
0.5 \\
0.4 \\
\end{array}
\]

\[
\begin{array}{c}
0.1 \\
0.6 \\
0.8 \\
0.5 \\
0.4 \\
0.6 \\
0.3 \\
0.3 \\
0.3 \\
0.4 \\
0.4 \\
\end{array}
\]

\[
\text{cost} = 0.1 + 0.6 + 0.8 + 0.5 + 0.3 + 0.4 = 2.7
\]
Object Ranking: Learning to Order Things [Cohen et al. 99]

- Since this is an NP-hard problem, it is solved heuristically.

\[
\begin{align*}
\text{Input:} & \quad \text{an instance set } X; \text{ a preference function } \text{PREF} \\
\text{Output:} & \quad \text{an approximately optimal ordering function } \hat{\rho} \\
\text{let} & \quad V = X \\
\text{for} & \quad \text{each } v \in V \text{ do} \\
\text{while} & \quad V \text{ is non-empty do} \\
\quad & \quad \pi(v) = \sum_{u \in V} \text{PREF}(v, u) - \sum_{u \in V} \text{PREF}(u, v) \\
\quad & \quad \text{let } t = \arg\max_{u \in V} \pi(u) \\
\quad & \quad \text{let } \hat{\rho}(t) = |V| \\
\quad & \quad V = V - \{t\} \\
\quad & \quad \text{for} \quad \text{each } v \in V \text{ do} \quad \pi(v) = \pi(v) + \text{PREF}(t, v) - \text{PREF}(v, t) \\
\text{endwhile}
\end{align*}
\]

- The algorithm successively chooses nodes having maximal „net-flow“ within the remaining subgraph.
- It can be shown to provide a 2-approximation to the optimal solution.
Label Ranking: Learning by Pairwise Comparison (LPC) [Hüllermeier et al. 2008]

Label ranking is the problem of learning a function \( X \rightarrow \Omega \), with \( \Omega \) the set of rankings (permutations) of a label set \( \mathcal{Y} = \{y_1, y_2, \ldots, y_k\} \), from exemplary pairwise preferences \( y_i \succ_x y_j \).

LPC trains a model

\[
\mathcal{M}_{i,j} : X \rightarrow [0, 1]
\]

for all \( i < j \). Given a query instance \( x \), this model is supposed to predict whether \( y_i \succ y_j \ (\mathcal{M}_{i,j}(x) = 1) \) or \( y_j \succ y_i \ (\mathcal{M}_{i,j}(x) = 0) \).

More generally, \( \mathcal{M}_{i,j}(x) \) is the estimated probability that \( y_i \succ y_j \).

Decomposition into \( k(k - 1)/2 \) binary classification problems.
## Label Ranking: Learning by Pairwise Comparison (LPC) [Hüllermeier et al. 2008]

Training data (for the label pair A and B):

<table>
<thead>
<tr>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>preferences</th>
<th>class</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>0</td>
<td>10</td>
<td>174</td>
<td>A ≻ B, B ≻ C, C ≻ D</td>
<td>1</td>
</tr>
<tr>
<td>1.45</td>
<td>0</td>
<td>32</td>
<td>277</td>
<td>D ≻ C</td>
<td></td>
</tr>
<tr>
<td>1.22</td>
<td>1</td>
<td>46</td>
<td>421</td>
<td>B ≻ D, B ≻ A, C ≻ D, A ≻ C</td>
<td>0</td>
</tr>
<tr>
<td>0.74</td>
<td>1</td>
<td>25</td>
<td>165</td>
<td>C ≻ A, C ≻ D, A ≻ B</td>
<td>1</td>
</tr>
<tr>
<td>0.95</td>
<td>1</td>
<td>72</td>
<td>273</td>
<td>B ≻ D, A ≻ D</td>
<td></td>
</tr>
<tr>
<td>1.04</td>
<td>0</td>
<td>33</td>
<td>158</td>
<td>D ≻ A, A ≻ B, C ≻ B, A ≻ C</td>
<td>1</td>
</tr>
</tbody>
</table>
At prediction time, a query instance is submitted to all models, and the predictions are combined into a binary preference relation:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>0.3</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>B</td>
<td>0.7</td>
<td></td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
<td>0.3</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>D</td>
<td>0.6</td>
<td>0.1</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>
At prediction time, a query instance is submitted to all models, and the predictions are combined into a binary preference relation:

From this relation, a ranking is derived by means of a ranking procedure. In the simplest case, this is done by sorting the labels according to their sum of weighted votes.
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Structured Output Prediction [Bakir et al. 2007]

- Rankings, multilabel classifications, etc. can be seen as specific types of **structured** (as opposed to scalar) **outputs**.
- Discriminative structured prediction algorithms infer a **joint scoring function on input-output pairs** and, for a given input, predict the output that maximises this scoring function.
- Joint feature map and scoring function

\[ \phi : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}^d, \quad f(x, y; w) = \langle w, \phi(x, y) \rangle \]

- The learning problem consists of estimating the weight vector, e.g., using structural risk minimization.
- Prediction requires solving a **decoding problem**:

\[ \hat{y} = \arg \max_{y \in \mathcal{Y}} f(x, y; w) = \arg \max_{y \in \mathcal{Y}} \langle w, \phi(x, y) \rangle \]
Structured Output Prediction [Bakir et al. 2007]

- **Preferences** are expressed through inequalities on inner products:

\[
\min_{w, \xi} \|w\|^2 + \nu \sum_{i=1}^{m} \xi_i
\]

s.t. \( \langle w, \phi(x_i, y_i) \rangle - \langle w, \phi(x_i, y) \rangle \geq \Delta(y_i, y) - \xi_i \) for all \( y \in Y \)

\( \xi_i \geq 0 \quad (i = 1, \ldots, m) \)

- The potentially huge number of constraints cannot be handled explicitly and calls for specific techniques (such as cutting plane optimization)
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Model-Based Methods for Ranking

- **Model-based approaches** to ranking proceed from specific assumptions about the possible rankings (*representation bias*) or make use of **probabilistic models** for rankings (parametrized probability distributions on the set of rankings).

- In the following, we shall see examples of both type:
  - Restriction to lexicographic preferences
  - Conditional preference networks (CP-nets)
  - Label ranking using the Plackett-Luce model
Learning Lexicographic Preference Models [Yaman et al. 2008]

- Suppose that objects are represented as feature vectors of length \( m \), and that each attribute has \( k \) values.
- For \( n = k^m \) objects, there are \( n! \) permutations (rankings).
- A **lexicographic order** is uniquely determined by
  - a total order of the attributes
  - a total order of each attribute domain
- **Example:** Four binary attributes \((m = 4, k = 2)\)
  - there are \( 16! \approx 2 \cdot 10^{13} \) rankings
  - but only \( (2^4) \cdot 4! = 384 \) of them can be expressed in terms of a lexicographic order
- [Yaman et al. 2008] present a learning algorithm that explicitly maintains the version space, i.e., the attribute-orders compatible with all pairwise preferences seen so far (assuming binary attributes with 1 preferred to 0). Predictions are derived based on the „votes“ of the consistent models.
Learning Conditional Preference (CP) Networks [Chevaleyre et al. 2010]

Compact representation of a partial order relation, exploiting conditional independence of preferences on attribute values.

Training data (possibly noisy):

(meat, red wine, Italian) > (veggie, red wine, Italian)
(fish, white wine, Chinese) > (veggie, red wine, Chinese)
(veggie, white wine, Chinese) > (veggie, red wine, Italian)
...
The Plackett-Luce (PL) model is specified by a parameter vector \( v = (v_1, v_2, \ldots, v_m) \in \mathbb{R}_+^m \):

\[
P(\pi | v) = \prod_{i=1}^{m} \frac{v_{\pi(i)}}{v_{\pi(i)} + v_{\pi(i+1)} + \cdots + v_{\pi(m)}}
\]

Reduces problem to learning a mapping \( x \mapsto v \).

Example: \( v = (1, 4, 2) \), \( P(\pi | v) = \frac{v_{\pi(1)}}{v_{\pi(1)} + v_{\pi(2)} + v_{\pi(3)}} \cdot \frac{v_{\pi(2)}}{v_{\pi(2)} + v_{\pi(3)}} \cdot 1 \)

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>0.0952</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>2</td>
<td>0.0476</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.1905</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0.0571</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0.3810</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0.2286</td>
</tr>
</tbody>
</table>
ML Estimation of the Weight Vector in Label Ranking

Assume \( \mathbf{x} = (x_1, \ldots, x_D) \in \mathbb{R}^D \) and model the \( v_i \) as log-linear functions:

\[
v_i = \exp \left( \sum_{d=1}^{D} \alpha_d^{(i)} \cdot x_d \right)
\]

Given training data \( \mathcal{T} = \{(\mathbf{x}^{(n)}, \pi^{(n)})\}_{n=1}^{N} \) with \( \mathbf{x}^{(n)} = (x_1^{(n)}, \ldots, x_D^{(n)}) \), the log-likelihood is given by

\[
L = \sum_{n=1}^{N} \sum_{m=1}^{M_n} \log \left( v(\pi^{(n)}(m), n) \right) - \log \sum_{j=m}^{M_n} v(\pi^{(n)}(j), n)
\]

where \( M_n \) is the number of labels in the ranking \( \pi^{(n)} \), and

\[
v(m, n) = \exp \left( \sum_{d=1}^{D} \alpha_d^{(m)} \cdot x_d^{(n)} \right).
\]

convex function, maximization through gradient ascent

can be seen as a log-linear utility function of i-th label
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Learning Local Preference Models [Cheng et al. 2009]

- Main idea of **instance-based (lazy) learning**: Given a new query (instance for which a prediction is requested), search for similar instances in a „case base“ (stored examples) and combine their outputs into a prediction.

- This is especially appealing for predicting **structured outputs** (like rankings) in a complex space \( Y \), as it circumvents the construction and explicit representation of a „\( Y \)-valued“ function.

- In the case of **ranking**, it essentially comes down to **aggregating** a set of (possibly partial or incomplete) rankings.
Finding the generalized median:

\[ \hat{y} = \arg \min_{y \in \mathcal{Y}} \sum_{i=1}^{k} \Delta(y_i, y) \]

- If Kendall’s tau is used as a distance, the generalized median is called the Kemendy-optimal ranking. Finding this ranking is an NP-hard problem (weighted feedback arc set tournament).
- In the case of Spearman’s rho (sum of squared rank distances), the problem can easily be solved through Borda count.
Learning Local Preference Models: Probabilistic Estimation

- Another approach is to assume the neighbored rankings to be generated by a **locally constant probability distribution**, to estimate the parameters of this distribution, and then to predict the mode [Cheng et al. 2009].

- For example, using again the PL model:

\[
P(\pi_1, \ldots, \pi_k | \boldsymbol{v}) = \prod_{j=1}^{k} P(\pi_j | \boldsymbol{v}) = \prod_{j=1}^{k} \prod_{i=1}^{m} \frac{v_{\pi(i)}}{v_{\pi(i)} + v_{\pi(i+1)} + \ldots + v_{\pi(m)}}
\]

\[
\log L = \sum_{j=1}^{k} \sum_{i=1}^{m} \log \left( v_{\pi(i)} \right) - \log \left( v_{\pi(i)} + v_{\pi(i+1)} + \ldots + v_{\pi(m)} \right)
\]

- Can easily be generalized to the case of incomplete rankings [Cheng et al. 2010c].
Summary of Main Algorithmic Principles

- **Reduction** of ranking to (binary) classification (e.g., constraint classification, LPC)
- **Direct optimization** of (regularized) smooth approximation of ranking losses (RankSVM, RankBoost, ...)
- **Structured output prediction**, learning joint scoring ("matching") function
- Learning parametrized **statistical ranking models** (e.g., Plackett-Luce)
- **Restricted model classes**, fitting (parametrized) deterministic models (e.g., lexicographic orders)
- **Lazy learning**, local preference aggregation (lazy learning)
References