Theorie des Algorithmischen Lernens
Sommersemester 2006

Teil 2.2: Lernen formaler Sprachen: Hypothesenräume

Version 1.1

Gliederung der LV

Teil 1: Motivation
1. Was ist Lernen
2. Das Szenario der Induktiven Inf erenz
3. Natürlichkeitsanforderungen

Teil 2: Lernen formaler Sprachen
1. Grundlegende Begriffe und Erkennungstypen
2. Die Rolle des Hypothesenraums
3. Lernen von Patternsprachen
4. Inkrementelles Lernen

Teil 3: Lernen endlicher Automaten

Teil 4: Lernen berechenbarer Funktionen
1. Grundlegende Begriffe und Erkennungstypen
2. Reflexion

Teil 5: Informationsextraktion
1. Island Wrappers
2. Query Scenarios

Different Approaches

When we have to learn an indexable class $\mathcal{L} = (L_j)_{j \in \mathbb{N}}$, we can choose the hypothesis space as follows:

1. use $\mathcal{L} = (L_j)_{j \in \mathbb{N}}$ as hypothesis space: ***exact*** identification
2. use another enumeration of $\mathcal{L} = (L_j)_{j \in \mathbb{N}}$ as hypothesis space: ***class preserving*** identification
3. use another indexable class $\mathcal{L}' = (L'_j)_{j \in \mathbb{N}}$ as hypothesis space that contains each L_j: ***class comprising*** identification

One could also ask for learnability w.r.t. *all* hypothesis spaces (***absolute*** learning)

$\longrightarrow$ until now, we considered class-comprising learning

- does it make a difference?

Learning in the Limit

Theorem 2.2.1:
Let $\mathcal{L} \in$ *LimTxt* and let $\mathcal{H}$ be *any class comprising* hypothesis space for $\mathcal{L}$. Then, there is an IIM M *LimTxt*$_{\mathcal{H}}$-identifying $\mathcal{L}$.

Proof.

Let M' be an IIM *LimTxt*$_{\mathcal{H}'}$-identifying $\mathcal{L}$.

$M(t_x)$:
If $M'(t_x) = ?$ then output "?".
Otherwise, set $j = M'(t_x)$ and test for $k = 0, \dots x$ whether or not

- $h_j(w) = h'_k(w)$ for all $w \in \Sigma^*$ with $|w| \leq x$.

If such a k has been found, output the least one, otherwise output "?".

Verification $\longrightarrow$ Exercise

Finite Learning

Theorem 2.2.2:
Let $\mathcal{L} \in$ *FinTxt* and let $\mathcal{H}$ be *any class preserving* hypothesis space for $\mathcal{L}$. Then, there is an IIM M *LimTxt*$_{\mathcal{H}}$-identifying $\mathcal{L}$.

Proof.

Let $\mathcal{L} \in$ *FinTxt*. By theorem 2.1.9 there are an indexing $\mathcal{L} = (L_j)_{j \in \mathbb{N}}$ and a recursively generable family $(T_j)_{j \in \mathbb{N}}$ of finite sets such that

- for all $j \in \mathbb{N}$, $T_j \subseteq L_j$
- for all $j, z \in \mathbb{N}$, if $T_j \subseteq L_z$ then $L_j = L_z$

$M(t_x)$:
If $x = 0$ or $M(t_{x-1}) =$ "?", goto (*). Otherwise output $M(t_{x-1})$.
(*) For $j = 0, 1, \dots, x$, generate T_j and test whether $T_j \subseteq t_x^+$.
If no such j has been found, output "?". Otherwise, let $\hat{j}$ be the minimal j and search for a j' such that $T_{\hat{j}} \subseteq h_{j'}$. Output j'.

Verification $\longrightarrow$ Exercise

Finite Learning

Theorem 2.2.3:
There is an $\mathcal{L} \in$ *FinTxt* and a class comprising hypothesis space $\mathcal{H}$ for $\mathcal{L}$ such that no IIM M *FinTxt*$_\mathcal{H}$-identifies $\mathcal{L}$.

Proof.

$\mathcal{L} = (L_j)_{j \in \mathbb{N}}$ with $L_j = \{a^j\}$. Clearly, $\mathcal{L} \in$ *FinTxt*.

Define $\mathcal{H}$ as follows:

$$h_{\langle k,x \rangle} = \begin{cases} \{a^k\} & : \phi_k(k) = x \\ \{a^k, b^{\phi_k(k)}\} & : \varphi_k(k) \downarrow \text{ and } \phi_k(k) \neq x \\ \{a^k\} & : \text{otherwise} \end{cases}$$

An IIM M *FinTxt*$_\mathcal{H}$-identifying $\mathcal{L}$ could be used to solve the halting problem:

On input k do:
Feed the text $a^k, a^k, a^k, \ldots$ to M until it outputs a hypothesis of form $\langle k, x \rangle$.
If $\phi_k(k) = x$, then output 1, otherwise output 0.

Verification $\longrightarrow$ Exercise

Conservative Learning

Theorem 2.2.4:
There is an $\mathcal{L}$ which can be conservatively learned, but only if the hypothesis space used is *class comprising.*

Theorem 2.2.5:
There is an $\mathcal{L}$ for wich

- there exists a class preserving hypothesis space $\mathcal{H}$ and an IIM M, such that M *ConsvTxt*$_{\mathcal{H}}$-identifies $\mathcal{L}$
- there exists a class preserving hypothesis space $\mathcal{H}'$ such that no IIM M *ConsvTxt*$_{\mathcal{H}'}$-identifies $\mathcal{L}$

proofs: see [2]

Summary

For learning in the limit:

- *exact, class preserving, class comprising, absolute class preserving, and absolute class comprising learning are of the same power*

For conservative learning:

- *absolute class preserving learning "$\subset$" class preserving learning "$\subset$" class comprising learning*

For finite learning:

- *absolute class preserving, class preserving, and class comprising learning are of the same power*
- *absolute class comprising learning "$\subset$" class comprising learning*

Changelog

- V1.1:
  - Folie 5: $\mathcal{L} \in \mathit{LimTxt} \longrightarrow \mathcal{L} \in \mathit{FinTxt}$