The Relationship Between PR & ROC Curves

Instructor: Frederik Janssen
Presenter: Wen Zhang
Outline

- Characters of ROC and PR curves
- The relationship between ROC and PR curves
  --- domination in ROC and PR space
  --- convex hull
  --- interpolation & AUC
  --- optimizing AUC
Evaluation of classifier performance

- Methods for evaluating the performance of classifiers
  --- simply accuracy
  --- ROC (recommended)
  --- PR (alternative to ROC curves)
ROC Curve

- Receiver Operation Characteristics (ROC): a technique for visualizing, organizing and selecting classifiers based on their performance
- Confusion Matrix
A Confusion Matrix is a table that is often used to describe the performance of a classification model. The table shows the number of correct and incorrect predictions broken down by each class. The rows of the table represent the actual outcomes, while the columns represent the predicted outcomes. The matrix is divided into four categories:

- **True Positives (TP)**: The model correctly predicted a positive outcome.
- **False Positives (FP)**: The model incorrectly predicted a positive outcome.
- **False Negatives (FN)**: The model incorrectly predicted a negative outcome.
- **True Negatives (TN)**: The model correctly predicted a negative outcome.

The matrix is typically used in binary classification problems, where there are two possible outcomes (e.g., disease presence or absence).
ROC Curve

- Class-decision Classify/Probabilistic Classify
- Classifier – (tpr, fpr) – point in ROC
- Property: without regard to class distributions or error costs (columnar ratio)
ROC and PR Curve

- **Classifier**: (classification model) : mapping from instances to predicted classes
- **ROC** (Receiver Operator Characteristic): trade-off between hit rates and false alarm rates
- **PR** (Precision-Recall): used in Information Retrieval, alternative to ROC, when difference are not apparent
ROC and PR Curve

- **ROC**: TPR/FPR
- **PR**: Precision/Recall
- **TPR** = Recall = \( \frac{TP}{TP+FN} \)  
  - “total positives”
- **FPR** = \( \frac{FP}{TN+FP} \)  
  - “total negatives”
- **Precision** = \( \frac{TP}{TP+FP} \)  
  - “predicted positives”
ROC and PR Curve

(a) Comparison in ROC space  
(b) Comparison in PR space
One-to-one correspondence between a curve in ROC space and a curve in PR space, if Recall ≠ 0 (FN retrieval)

Under the fixed number of positive and negative examples, domination in ROC and PR
Proof.

Proof “

Suppose: Curve I dominates curve II in ROC space but not in PR space

point B on Curve I & point A on Curve II
with TPR_A = TPR_B
Domination in ROC \( \Rightarrow FPR_A \geq FPR_B \)
PR space, Rec(A) = Rec(B)

Assumption: Prec(A) > Prec(B)
Domination in ROC space

(a) Case 1: $FPR(A) > FPR(B)$

(b) Case 2: $FPR(A) = FPR(B)$
Proof.

- Domination in ROC \( \text{FPR}_A \geq \text{FPR}_B \) with fixed \( N \) \( \text{FP}_A \geq \text{FP}_B \)
- \( \text{TPR}_A = \text{TPR}_B \) with fixed \( P \) \( \text{TP}_A = \text{TP}_B \)

Precision A = \( \frac{\text{TP}_A}{\text{FP}_A + \text{TP}_A} \)
Precision B = \( \frac{\text{TP}_B}{\text{FP}_B + \text{TP}_B} \)

\[ \text{Prec}(A) \leq \text{Prec}(B) \quad \text{CONFLICT!!} \]

So Curve I should also dominate Curve II in PR space.
Proof.

- Proof “$\Rightarrow$“ : analog
  
  So Curve I should also dominate Curve II in ROC space.

- A curve dominates in ROC space if and only if it dominates in PR space.
Convex Hull

- Convex Hull is a set of points in ROC with following three criteria:
  2. Linear interpolation between adjacent points
  3. No points above the final curve
  4. Any points connection lines equal or under the C.H.
C.H. & achievable PR curve

(a) Convex hull in ROC space
(b) Curves in ROC space
(c) Equivalent curves in PR space
“Convex Hull” in PR space

- Convex Hull in ROC \textarrowachievable PR curve
- Achievable PR curve: \textcolor{red}{non-linear interpolation}
  
  (FP replaces FN in the denominator of the Precision metric)
convex hull

- Method to build convex hull in Roc is on hand.
- How to construct an achievable PR curve?
Interpolation & AUC

- Linear interpolation by ROC curve, but non-linear interpolation by PR curves
- Solution: interpolation in ROC space

PR curve
infinitely many points?
Interpolation in PR curve (Goadrich 2004)

**Example:** two points, A and B => points between A and B

**Method:** “1 positive vs. n negatives “ with 

\[ n = \frac{(FP_B - FP_A)}{(TP_B - TP_A)} \]  

“local skew”, \( 1 \leq x \leq TP_B - TP_A \)

new points:

\[
\left( \frac{TP_A + x}{\text{Total Pos}}, \frac{TP_A + x}{TP_A + x + FP_A + \frac{FP_B - FP_A}{TP_B - TP_A} x} \right)
\]
Example

<table>
<thead>
<tr>
<th></th>
<th>TP</th>
<th>FP</th>
<th>REC</th>
<th>PREC</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>5</td>
<td>0.25</td>
<td>0.500</td>
</tr>
<tr>
<td>.</td>
<td>6</td>
<td>10</td>
<td>0.30</td>
<td>0.375</td>
</tr>
<tr>
<td>.</td>
<td>7</td>
<td>15</td>
<td>0.35</td>
<td>0.318</td>
</tr>
<tr>
<td>.</td>
<td>8</td>
<td>20</td>
<td>0.40</td>
<td>0.286</td>
</tr>
<tr>
<td>.</td>
<td>9</td>
<td>25</td>
<td>0.45</td>
<td>0.265</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>30</td>
<td>0.50</td>
<td>0.250</td>
</tr>
</tbody>
</table>
AUC-ROC & AUC-PR

- AUC–ROC
  trapezoidal areas under curve
- AUC-PR
Optimizing AUC

Are those Algorithm for optimizing AUC-ROC also help to improve AUC-PR?
example

- 20 positives and 2000 negatives, result:
  AUC-ROC 0.813(I);
  0.875(II) II wins
  AUC-PR 0.514(I);
  0.038(II) I wins

- A lower Recall rage with higher Precision is required by AUC in PR.
Conclusion

- Same points contained in ROC curve and PR curve (correspondent)
- Convex hull in ROC vs. achievable PR curve
- Non-linear interpolation in PR space
- Those algorithms to optimize AUC-ROC doesn’t guarantee to optimize AUC-PR
Thanks for attention!

Questions?