The Relationship Between PR & ROC Curves

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Outline

- Characters of ROC and PR curves
- The relationship between ROC and PR curves

--- domination in ROC and PR space

--- convex hull

--- interpolation & AUC

--- optimizing AUC

Evaluation of classifier performance

- Methods for evaluating the performance of classifiers

--- simply accuracy

--- ROC (recommended)

--- PR (alternative to ROC curves)

ROC Curve

- Receiver Operation Characteristics (ROC):

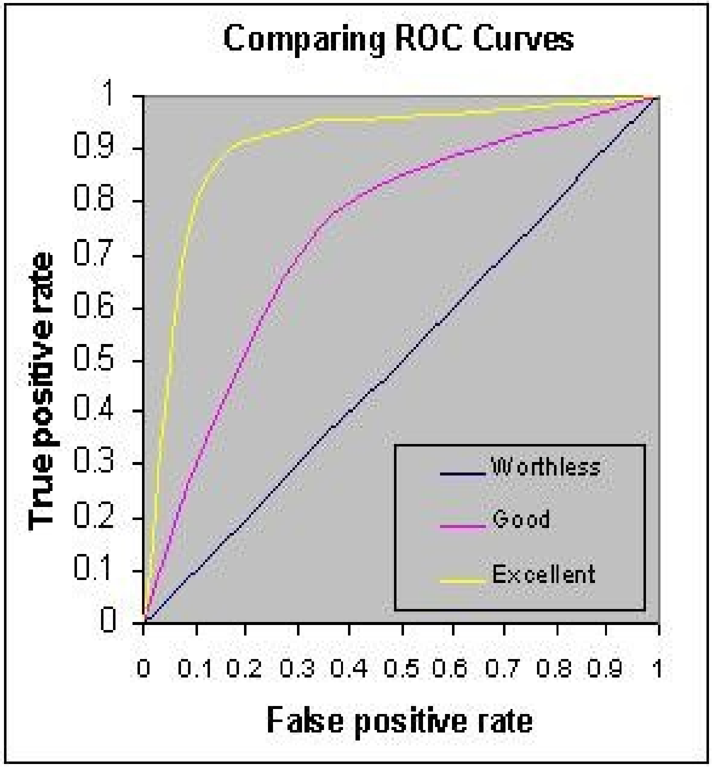
  a technique for visualizing, organizing and selecting classifiers based on their performance
- Confusion Matrix

Confusion Matrix

	True Class P	*True Class* N
Hypothesized Class Y	**T**rue **P**ositives	**F**alse **P**ositives
Hypothesized Class N	**F**alse **N**egatives	**T**rue **N**egatives

ROC Curve

- Class-decision Classify/ Probabilistic Classify
- Classifier – (tpr, fpr) – point in ROC
- Property: without regard to class distributions or error costs (columnar ratio)



ROC and PR Curve

- **Classifier**(classification model) : mapping from instances to predicted classes
- **ROC** (Receiver Operator Characteristic): ***trade-off*** between hit rates and false alarm rates
- **PR** (Precision-Recall): used in Information Retrieval,

alternative to ROC, when difference are not apparent

ROC and PR Curve

- **ROC** : TPR/FPR
- **PR** : Precision/Recall
- **TPR=Recall**= TP/(TP+FN)

  " total positives"

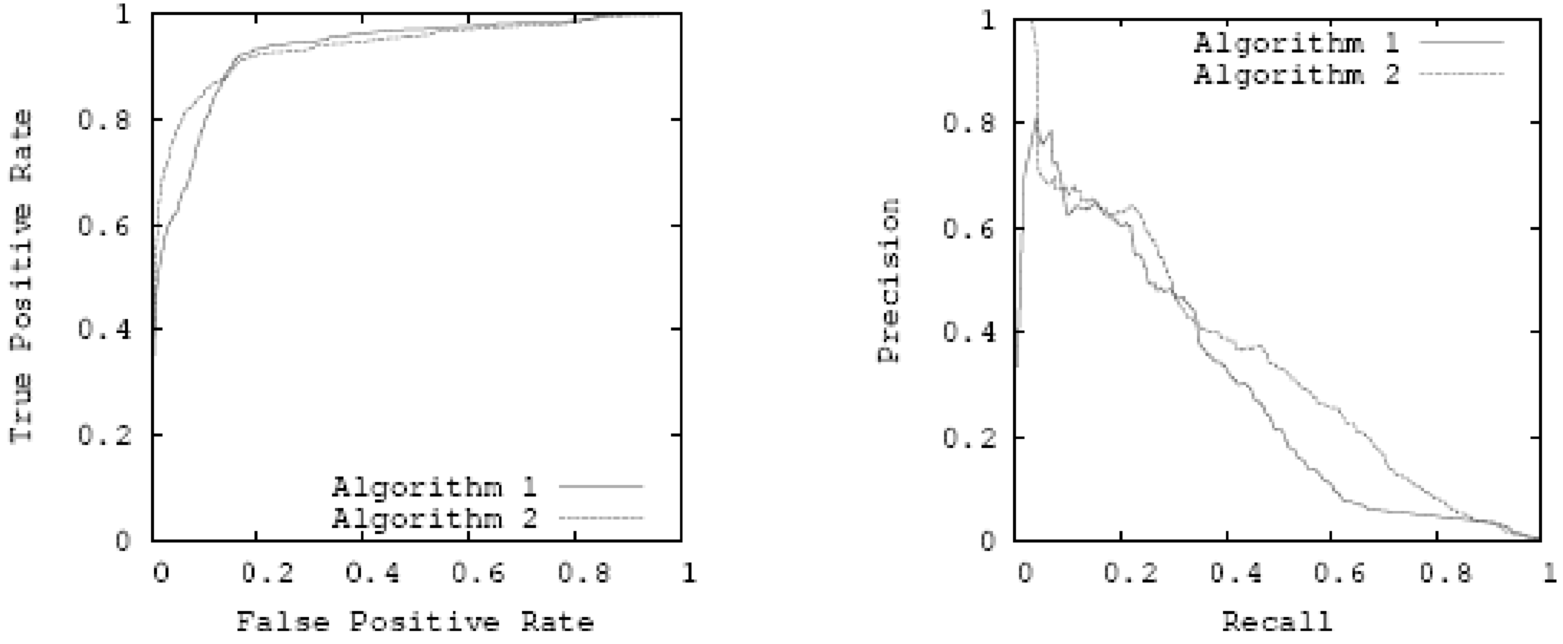
- **FPR**= FP/(TN+FP)

  " total negatives"

- **Precision**= TP/(TP+FP)

  "predicted positives"

ROC and PR Curve



(a) Comparison in ROC space

(b) Comparison in PR space

Domination in ROC & PR

- One-to-one correspondence between a curve in ROC space and a curve in PR space , if Recall ≠ 0 (FN retrieval)

(ROC ⟺ Confusion Matrix ⟺ PR)

- Under the fixed number of positive and negative examples, domination in ROC ⟺ domination in PR

Proof.

- Proof “$\Rightarrow$“

Suppose: Curve I dominates curve II in ROC space but not in PR space

point B on Curve I & point A on Curve II

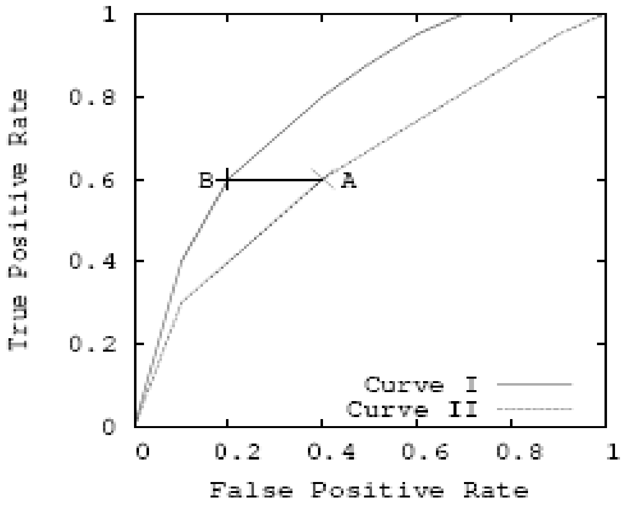
with $TPR_A = TPR_B$

Domination in ROC $\Rightarrow$ $FPR_A \geq FPR_B$

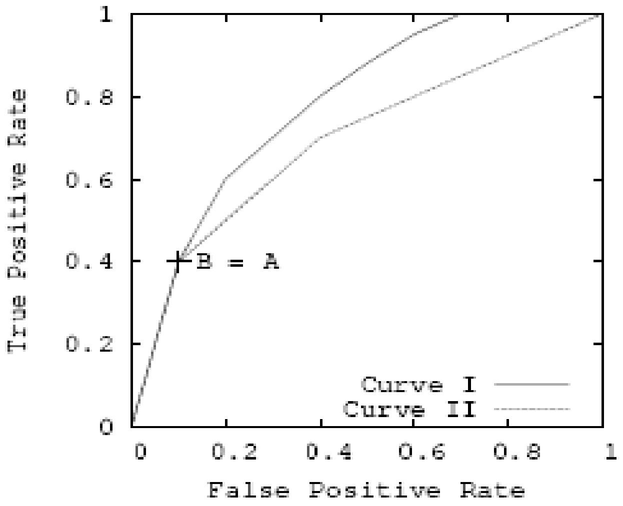
PR space, Rec(A)=Rec(B)

<u>Assumption</u>: Prec(A) > Prec(B)

Domination in ROC space



(a) Case 1: $FPR(A) > FPR(B)$



(b) Case 2: $FPR(A) = FPR(B)$

Proof.

- Domination in ROC → $FPR_A \geq FPR_B$ with fixed N → $FP_A \geq FP_B$
- $TPR_A = TPR_B$ → with fixed P → $TP_A = TP_B$

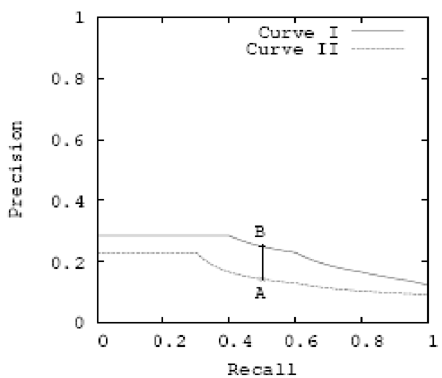
Precision A = $TP_A/(FP_A+TP_A)$

Precision B = $TP_B/(FP_B+TP_B)$

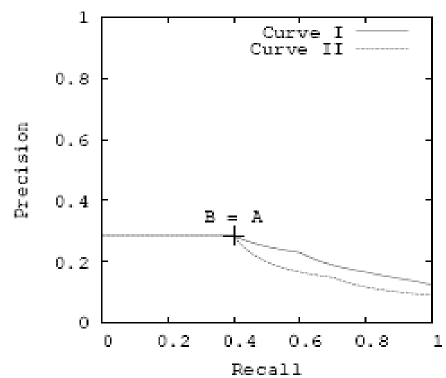
→ $Prec(A) \leq Prec(B)$ CONFLICT!!

<u>So Curve I should also dominate Curve II in PR space.</u>

Proof.



(a) Case 1: $PRECISION(A) < PRECISION(B)$



(b) Case 2: $PRECISION(A) = PRECISION(B)$

- Proof “ ⇐ “ : analog

So Curve I should also dominate Curve II in ROC space.

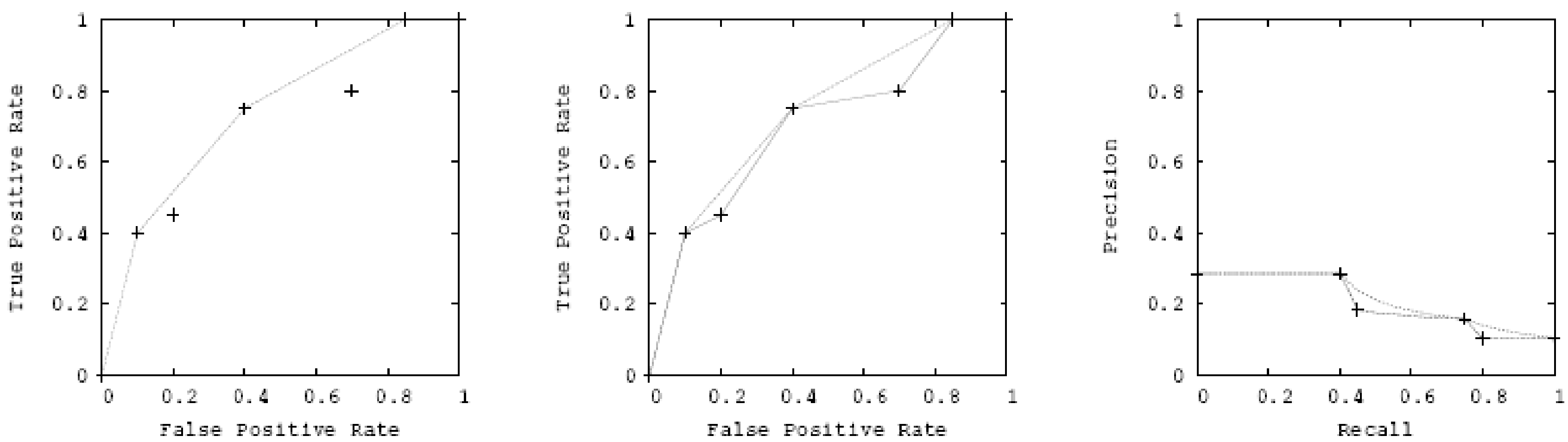
- A curve dominates in ROC space if and only if it dominates in PR space.

Convex Hull

- Convex Hull is a set of points in ROC with following three criteria:

2. Linear interpolation between adjacent points
3. No points above the final curve
4. Any points connection lines equal or under the C.H.

C.H. & achievable PR curve



(a) Convex hull in ROC space (b) Curves in ROC space (c) Equivalent curves in PR space

“Convex Hull” in PR space

- Convex Hull in ROC ⟺ achievable PR curve
- Achievable PR curve: non-linear interpolation

(FP replaces FN in the denominator of the Precision metric)

convex hull

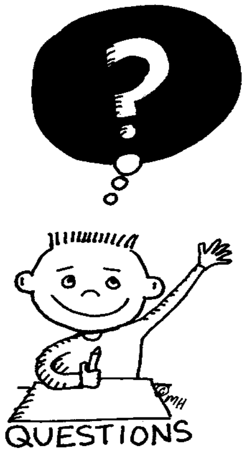
- Method to build convex hull in Roc is on hand.
- How to construct an achievable PR curve?

Interpolation & AUC

- Linear interpolation by ROC curve, but non-linear interpolation by PR curves
- Solution: interpolation in ROC space

PR curve

infinitely many points?



Interpolation in PR curve (Goadrich 2004)

Example: two points, A and B => points between A and B

Method : "1 positive vs. n negatives " with

n = $(FP_B - FP_A)/(TP_B - TP_A)$ "local skew", $1 \le x \le TP_B - TP_A$

new points:

$$\left(\frac{TP_A + x}{\text{Total Pos}}, \frac{TP_A + x}{TP_A + x + FP_A + \frac{FP_B - FP_A}{TP_B - TP_A} x} \right)$$

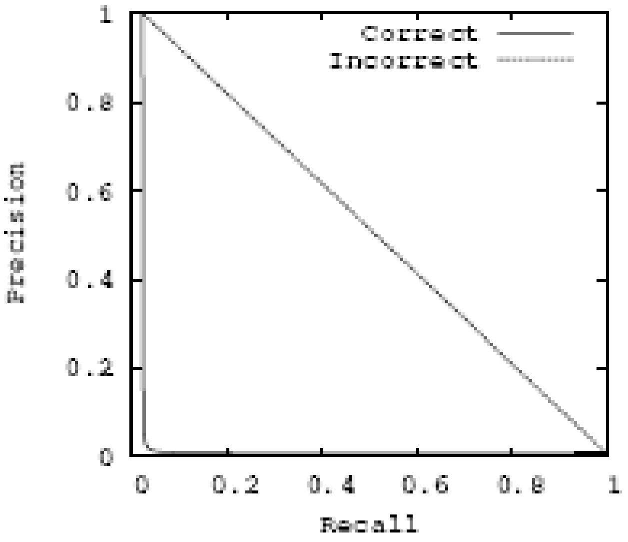
Example

	TP	FP	REC	PREC
A	5	5	0.25	0.500
.	6	10	0.30	0.375
.	7	15	0.35	0.318
.	8	20	0.40	0.286
.	9	25	0.45	0.265
B	10	30	0.50	0.250

AUC-ROC & AUC-PR

- AUC–ROC

  trapezoidal areas under curve
- AUC-PR



Optimizing AUC

Are those Algorithm for optimizing AUC-ROC also help to improve AUC-PR?

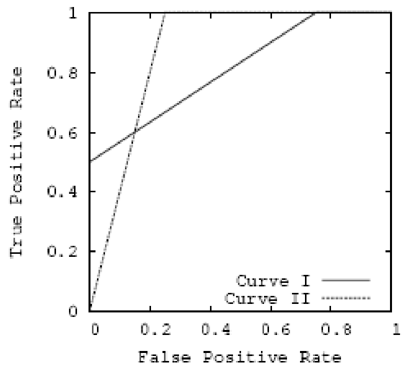
example

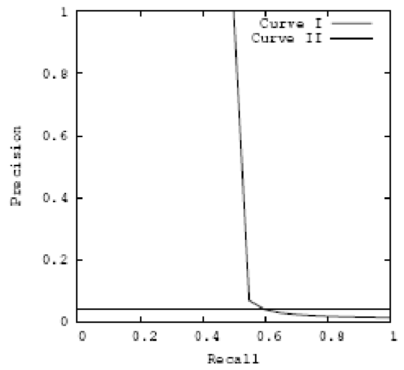
- 20 positives and 2000 negatives, result :

  AUC-ROC 0.813(I); 0.875(II) II wins

  AUC-PR 0.514(I); 0.038(II) I wins

  - A lower Recall rage with higher Precision is required by AUC in PR.





Conclusion

- Same points contained in ROC curve and PR curve (correspondent)
- Convex hull in ROC vs. achievable PR curve
- Non-linear interpolation in PR space
- Those algorithms to optimize AUC-ROC doesn’t guarantee to optimize AUC-PR

Thanks for attention!

Questions?